Homework 3

1. Answers
   1. 2.5
   2. 0.502
   3. 0.83
2. Answers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| y | x | | | | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | Py(y) |
| 0 | 0.040 | 0.010 | 0.025 | 0 | 0.025 | 0 | **0.1** |
| 1 | 0.200 | 0 | 0.050 | 0.300 | 0 | 0 | **0.55** |
| 2 | 0.100 | 0.100 | 0 | 0 | 0 | 0.150 | **0.35** |
| Px(x) | **0.34** | **0.11** | **0.075** | **0.3** | **0.025** | **0.15** | 1 |

* 1. Because Px(x) and Py(y) =1 that means this is a valid joint probability distribution
  2. See bottom row of table (bolded)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 4 | 5 |
| F(x|y=0) | 0.4 | 0.1 | 0.25 | 0 | 0.25 | 0 |
| F(x|y=1) | 0.36 | 0 | 0.09 | 0.5454 | 0 | 0 |
| F(x|y=2) | 0.285 | 0.285 | 0 | 0 | 0 | 0.428 |

* 1. See last column of table (bolded)
  2. See table above
  3. 0.6
  4. They are dependent, see work in the how I solved section

1. Answers
   1. 4
   2. 0.04
   3. 2x, 0 ≤ x ≤ 1 (marginal density of x)
   4. 2y, 0 ≤ y ≤ 1 (marginal density of y)
   5. 2x, , 0 ≤ x ≤ 1 (f(x|y))
   6. 2y, , 0 ≤ y ≤ 1 (f(y|x)
   7. 2x, , 0 ≤ x ≤ 1 (f(x|y=0.5))
   8. Independent

# How I solved:

1. Density Function (Continuous Random Variables)
   1. We know that the density function integrated with infinity and negative infinity as the bounds should equal 1. So we integrate cx^4 and set it equal to 1 with the bounds given to us (-1 to 1)
      1. . Which gives us 5/2 or 2.5
   2. =
      1. So now we have F(x) = 0 if x ≤ -1, if -1 < x < 1, and 1 if x ≥ 1.
   3. To solve F(1/3) plug 1/3 in for x which gives you 0.502
   4. |x| means we would have to integrate f(x) from -1 to -0.7 added to the integration of f(x) from 0.7 to 1, however, P(|x|>0.7) is equivalent to F(0.7)\*2 which is 0.83
2. Joint Probability Distribution

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| y | x | | | | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | Py(y) |
| 0 | 0.040 | 0.010 | 0.025 | 0 | 0.025 | 0 | 0.1 |
| 1 | 0.200 | 0 | 0.050 | 0.300 | 0 | 0 | 0.55 |
| 2 | 0.100 | 0.100 | 0 | 0 | 0 | 0.150 | 0.35 |
| Px(x) | 0.34 | 0.11 | 0.075 | 0.3 | 0.025 | 0.15 | 1 |

1. Looked at table
2. Add whats in the columns together
3. Add whats in the rows together
4. F(x|y) = f(x,y)/f(y), plug and chug
5. P(X > 0 | Y =0) i.e for all values of X greater than 0 given Y = 0
   1. P(X=1,2,3,4,5, Y = 0)/P(Y=0)
   2. 0.01+0.025+0.025/0.1=0.6
6. Independence defined as p(x,y) = px(x)py(y)
   1. So we’ll take P(2,2) = 0.075\*0.35 = 0.026 but it should be 0 to be independent, so the variables are dependent
   2. As a sanity check we’ll look at P(1,0) = 0.11\*0.1= 0.011 but should be 0.010
7. Joint Density
   1. Take double integral with limits being 0 to 1 for both x and y
      1. Gives you 1/4. So we have c1/4 =1 which makes c = 4
   2. P(x < 0.4, Y < 0.5)
      1. Take the double integral of 4xy with the limits 0 to 0.5 for y and 0 to 0.4 for x
   3. Take the integral of f(x,y) with the limits of 0 to 1 with respect to y
   4. Do the same as did to find marginal density of X but with respect to x
   5. F(y|x) = f(x,y)/f(x)
   6. F(x|y) = f(x,y)/f(y)
   7. F(x|y=0.5) = f(x, 0.5)/f(y) = 4x(0.5)/2(0.5)
   8. f(x,y) = fx(x)fy(y)
      1. 4xy = 2x\*2y = 4xy, which means independent